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Uncertainty and Risk Around Pakistan's Debt Sustainability: A Fan Chart Approach

Samar Ali[1], Dr. Tahseen Iqbal[2], Dr. Ambreen Fatima[3]

[1]Assistant Professor Shah Abdul Latif University Ghotki Campus. Samar.ghanghro@gmail.com
[2]Assistant Professor Applied Economic Research Centre (AERC) University of Karachi. tehseen.iqbal@hotmail.com
[3]Associate Professor Applied Economic Research Centre (AERC) University of Karachi. amber_aerc@yahoo.com

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Corresponding Author: Samar Ali
Email: samar.ghanghro@gmail.com

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ABSTRACT

The debt sustainability has emerged as a critical challenge for many emerging economies, including Pakistan. The fiscal response function (FRF), is the framework to focus on the government's capability to adjust its fiscal position in react to increasing debt levels. This study fills a critical gap in the current literature by integrating the idea of the primary expenditure gap, demarcated as the deviation of actual primary expenditure from its mean expenditure level. This research paper focuses on fiscal imbalances in Pakistan, which are mostly expenditure-driven, and ignoring this element may lead to lacking or ambiguous conclusions regarding debt sustainability in Pakistan. This research models the primary balance as a function of total public debt, GDP gap, primary expenditure gap and a democracy dummy variable to capture macroeconomic and institutional effects on fiscal policy behavior through ARDL econometric model for the time period of 1976 to 2024. Furthermore, the study employs a fan chart forecasting approach to forecast total public debt, domestic debt, and external debt, incorporating uncertainty and risk into future debt paths. The FRF approach provides an organized method to assess whether fiscal policy in Pakistan gratifies the sustainability condition, while the primary balance responds positively to an increase in public debt. The empirical results of study evaluate whether Pakistan demonstrate a stabilizing fiscal response to public debt and examines the role of cyclical conditions, captured by the GDP gap, primary expenditure gap as well as political regimes insight through the democracy dummy. The study offers important policy implications for designing effective fiscal consolidation strategies, emphasizing the need to manage primary expenditure alongside improving fiscal responsiveness to ensure long-term debt sustainability in Pakistan.

INTRODUCTION

Sustainability of the public debt has emerged as a dominant issue of macroeconomic stability, especially in third-world economies like Pakistan, where structural inefficiencies, low revenue mobilization and chronic fiscal deficit have resulted in a cumulative buildup of the public debt. In the last several decades, both domestic and external debt have dramatically increased in Pakistan, and the question of whether the government can afford to fulfill the debt without damaging economic growth and, therefore, using the destabilizing fiscal measures, has become an acute issue. Public debt sustainability is not just a fiscal problem, but a macroeconomic problem that has an impact on investment, growth and financial stability Ali, S., et.al (2024). Thus debt sustainability remained a controversial debate in the upper and lower house of parliament in Pakistan over the past few decades. Considering the issue, the Pakistan's government passed "Fiscal Responsibility and Debt Limitation (FRDL) act 2005" through the parliament to overcome the debt sustainability issue. Later on it was amended in 2017 and in 2022. Debt sustainability is more often described as the possibility of the government to continue with its current borrowing trend without necessarily having to change its policy abruptly or face default on its commitments [Slany, A. (2020)]. The theoretical basis of debt sustainability is the intertemporal budget constraint which states that the stock of current debt should be equivalent to the present value of future surpluses of primary. In this context, the fiscal balance, the balance without interest payments, is predominant since it shows the discretionary fiscal effort of the government in stabilizing the dynamics of debt. A continuing primary deficit will show that the government is not just borrowing to pay interest but also to meet its non-interest spending, which will add to the debt accumulation [Uctum, M., & Wickens, M. (1997), Bohn, H. (1998), Arnone, M., & Presbitero, A. F. (2007), Contessi, S. (2012), Tielens, J., et al (2014), Mahmood, T., et al (2014), Adler, G., & Sosa, S. (2016), Kaur, B., et al (2018), and Chandia, K. E., et al (2019)].

According to Bohn, H. (1998), Ghosh, A. R., et al (2013), Bouabdallah, O., et al (2017), Ejaz, M., & Hyder, K. (2019) and other scholars the fiscal response function is one of the most popular empirical methods to analyze the sustainability of debts as they analyze the response of the primary balance to the increase or decrease of the public debt. By this definition, fiscal policy sustainable when the government raises its primary surplus as debt levels rise. Empirical research on Pakistan has used this method, and the result have shown a positive correlation between the

primary balance and lagged debt, indicating of weak-form sustainability, but the response is usually too weak to ensure long-term stability. Likewise, studies show that the fiscal policy changes in Pakistan are affected by debt dynamics, the main balance reacts to the changes in the debt levels, although with a limited response. Although this is the case, a weak fiscal policy in Pakistan has been characterized by structural deficiencies such as a history of deficits and dependence on borrowing to meet government spending commitments. The high rate of an increase in the public debt especially over the past two decades, has brought into question the capacity of the country to maintain its fiscal stance. Empirical studies indicate that the growth rate of debt in Pakistan has been growing much higher than the level of economic growth, thus straining the sustainability of fiscal policy as well as the macroeconomic stability. Such an imbalance between growth and accumulation of debt raises the concern that the underlying fiscal mechanisms warrant more detail exploration. Macroeconomic conditions along with the level of debt, are also important factors that influence fiscal policy behavior. A significant variable here is the GDP gap, which indicates the difference between the actual output and the potential level of output. According to economic theory, governments are supposed to implement counter-cyclical fiscal policies, that is, have a deficit in recession and a surplus when the economy is expanding. Nonetheless, emerging markets tend to have pro-cyclical fiscal policy because they have small fiscal space as well as institutional limitations. By including the GDP gap in the fiscal response functional, it is possible to have a clearer insight into the effect of cyclical conditions on fiscal adjustment and debt sustainability [Zdravković, A., et al (2013), Plödt, M., & Reicher, C. (2014), Lau, E., & Syn-Yee, A. L. (2018) and Iorio, F. D., & Fachin, S. (2022)].

Moreover, political and institutional aspects are becoming more and more recognized as critical determinants of fiscal policy. In the instance of Pakistan, democratic and non-democratic rule have been linked to various fiscal performances. In order to capture these dynamics, this paper includes a dummy variable of democracy separate the democratic and non-democratic regimes [Rogoff, (1990), Brender & Drazen, (2005); and Alt, J. E., & Lassen, D. D. (2006)]. The rationale behind the presence of this variable is that democratic regimes will be under more pressure to spend more money on the electoral considerations, which can lead to weaker fiscal discipline. On the other hand, non-democratic regimes can be characterized by tight fiscal control and lack of transparency and accountability [Ali, S, et.al (2022)]. Further the impact of institutional quality and political variables in the formation of fiscal behavior and debt outcomes are also emphasized

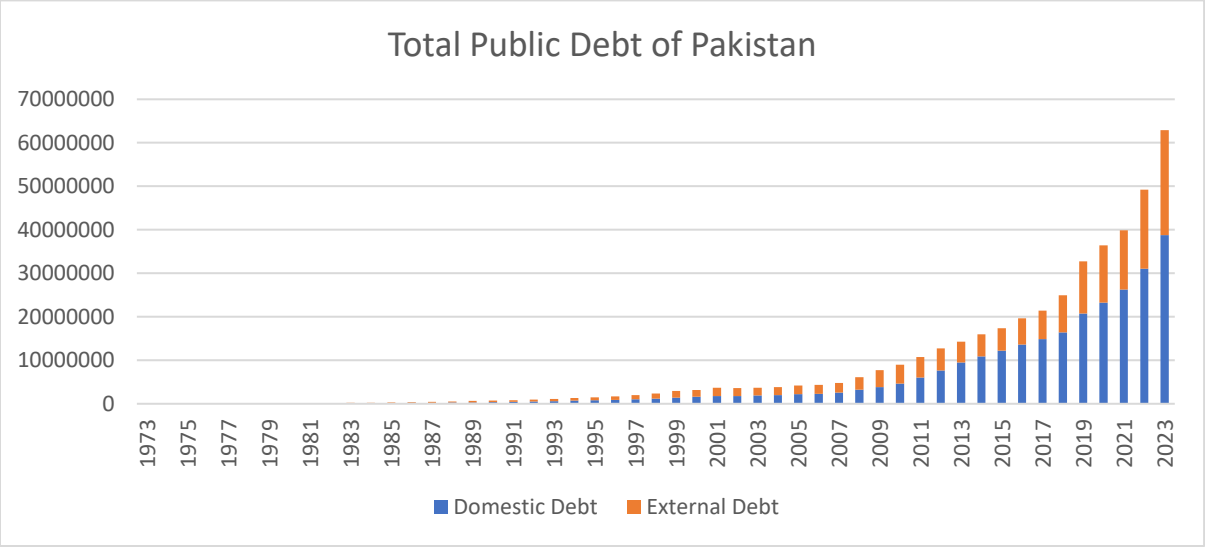
in existing literature. Although there have been tremendous advancements in the literature on debt sustainability in Pakistan, most literature is focused on aggregate fiscal measures, especially the primary balance, without sufficiently delving into the constituents of the primary balance. The researchers like Chandia, K. E., & Javid, A. Y. (2013), Mansoor, S., et.al (2020), Fatemah, A. (2024) and others takes the primary balance as a univariate, which is implicitly assuming that the fiscal changes are made equally in terms of revenue and expenditure. Empirical evidence however, indicates that this assumption might not be true in the case of Pakistan. Indeed, in many cases, fiscal adjustments are motivated more by the fiscal adjustments in the government spending than by the enhancement in revenue collection. Indicatively, empirical research indicate the government spending play a negative role on the primary balance, whereas revenue changes are comparatively feeble. This fact indicates a critical but previously under-researched aspect of fiscal policy analysis, the role of primary expenditure. Primary expenditure, which is spending by the government without payment of interest, is a key element of fiscal policy, and it has a direct effect on the debt dynamics. In Pakistan, the public spending is highly tilted towards current expenditure which consists of subsidies, governmental expenditures and military spending while development spending is comparatively limited. Furthermore, the increasing debt servicing costs have also diminished fiscal space, crowding out productive spending, and increased fiscal imbalances.

Although the role of primary expenditure is crucial, but it has not been included in the previous framework of fiscal response functions in existing literature on Pakistan. This is a major gap filled in this research. Although past literature has concentrated on how the primary balance is responsive to debt, it has not been discussed the extent to which the government expenditure levels are consistent with sustainable dynamics of debt processes or whether the level of deviation in primary expenditure leads to fiscal instability. That is, the literature does not examine the major element of debt sustainability like PE_C, the gap between the actual primary expenditure and the estimated amount of primary expenditure and it's consistent with debt sustainability. This gap is especially noteworthy in the case of Pakistan, where fiscal imbalances are mostly expenditure-related. The lack of information on primary expenditure gap prevents the designing of effective fiscal consolidation measures by the policymakers. Lack of knowledge on sustainability of government expenditure can result into policy responses that concentrate on the short-term elements of reducing the deficit instead of the long-term fiscal stability.

Thus, the main goal of the paper is to explore the debt sustainability of Pakistan within the framework of a fiscal response function that includes the key macroeconomic and institutional variables, such as primary balance, total public debt, GDP gap, and a democracy dummy. More to the point, the research takes primary expenditure as a gap in the current literature, which needs to be addressed this integrate the expenditure dynamics in the analysis of fiscal sustainability.

Further this paper used modern fan chart to visualize the most likely path of the point estimate with entire probability occurrence rang of the forecasted points. The research makes a contribution to the literature in various aspects. First, it enhances the traditional fiscal response role by incorporates macroeconomic and political variables that are applicable in the Pakistan context. Second, it highlights the need to disaggregate fiscal variables with primary expenditure being a determinant of debt sustainability. Third, it visualizes the forecast of debt sustainability of three variables total public debt percentage of GDP (TPD), Domestic debt percentage of GDP (DD), and External Debt percentage of GDP (ED). Lastly, it gives policy-relevant information on how fiscal strategies can be designed to attain sustainable debt levels. In sum, though the available literature is very insightful in the context of the debt sustainability in Pakistan, without a thorough analysis of the primary expenditure, it still lacks comprehensiveness. The current study will fill in this gap by providing a more detailed insight on how fiscal policy is conducted and what it means to long-term debt sustainability.

Figure: 1.1: Total Public Debt of Pakistan



Source: The Economic survey and its statistical supplements 2005-06, 2013-14, 2023-24

LITERATURE REVIEW

Debt sustainability has become one of the most important issues in the modern macroeconomic policy discussion. The increasing accumulation rate of public debt in both developed and developing economies, especially after the global financial crisis of 2008 and the COVID-19 pandemic, has made the academic and policy analyses of the circumstances under which debt paths can be kept manageable without causing a fiscal crisis or a sovereign default more analytical of the situation. These theoretical thresholds, empirical determinants, and policy implications of debt sustainability have been the subject of debate among scholars, producing a rich body of literature spanning multiple methodological traditions and regional settings (Reinhart & Rogoff, 2010; Blanchard et al., 2021).To Pakistan, a nation that has been heavily dependent on external and internal borrowing to fund its long-standing fiscal deficits, the issue of debt sustainability is not simply an academic one- it is an issue of existential economic relevance (Arby & Hussain, 2012; Chaudhry et al., 2020).

The classical literature on the sustainability of debt is based on the intertemporal budget constraint, which states that the present discounted value of future primary surpluses equals the outstanding public debt. This condition was among the first to be formalised econometrically, with a fiscal reaction function in which governments increase primary balances in response to an increase in the debt-to-equity ratio. The model that he created has served as a standard against which other empirical studies have been evaluated. Expanding on this concept, Blanchard (1990) proposed the fiscal gap, the adjustment in taxation required to stabilise debt over a specified time horizon, thus bridging the gap between theoretical constructs and practical policy analysis. Reinhart and Rogoff (2010) added to the theoretical debate by arguing that high levels of public debt are related to lower economic growth, especially beyond a threshold of 90 percent of GDP, a statement that sparked enormous academic debate. Later research by Herndon et al. (2014) found that in the analysis by Reinhart and Rogoff, there were coding mistakes and methodological issues that led to the conclusion that the empirical relationship between debt and growth is significantly more nuanced and context-dependent. Nevertheless, the hypothesis of the debt overhang, that excessive debt discourages investment and undermines future growth prospects remain alive in policy debates at the International Monetary Fund (IMF) and the World Bank. Theoretical background Buiter (1985) laid theoretical groundwork by breaking down the intertemporal

solvency condition of the government in observable flows of fiscal policy, and Chalk and Hemming (2000) extended this to stochastic environments, showing that sustainability tests must take into account uncertainty in interest rates and growth differentials. In a more modern approach, Blanchard et al. (2021) re-examined the debt-growth nexus in a low-interest-rate environment, arguing that when the risk-free rate persistently falls below the growth rate, a traditional burden of public debt can be greatly relieved. This has significant implications on the sustainability thresholds, but Mehrotra and Sergeyev (2021) warns that these benign conditions may not be sustainable in the long run and it is highly sensitive to the dynamics of global financial markets. D'Erasmo et al. (2016) provided a political economy dimension, opining that the sustainability of debt is not a technical question but heavily subject to the incentives of governments to repay debt, the quality of institutions and the form of domestic and foreign debt. This literature is especially applicable in situations like Pakistan where political instability and institutional weaknesses meet with fiscal pressures in intricate combinations Zaidi, (2015); Hyder and Shah, (2004).

There is a considerable amount of empirical literature that has studied the macroeconomic factors that dictate whether or not the debt path of a particular country can be considered sustainable. The core determinants have always been identified as GDP growth, interest rates, primary fiscal balances, exchange rate dynamics and external sector conditions. Escolano (2010) presented a commonly used decomposition model indicating that changes in the debt-to-GDP ratio could be attributed to the primary balance, the interest-growth differential, and stock-flow adjustments, such as exchange rate valuation effects. This framework has been operationalized in IMF Debt Sustainability Analyses (DSAs) of both advanced and emerging market economies. Eberhardt and Presbitero (2015) used panel data techniques to re-estimate the debt-growth relationship across 118 countries and found substantial heterogeneity in the relationship between debt and growth across country groupings. It is also because the estimator they used (the Common Correlated Effects, or CCE, estimator) enabled them to control for cross-sectional dependence, a methodological improvement that previous pooled OLS studies had failed to consider. On the same note, Pescatori et al. (2014) did not find any specific threshold beyond which debt is universally detrimental to economic growth, but instead it was found that the direction of debt, whether upward or downward, was more important than the actual level of debt to the economy over the medium term. Aghion et al. (2009) have tested the role of exchange rate regimes in debt sustainability and found that flexible exchange rates can serve as automatic stabilizers, reducing susceptibility to

external shocks. For countries with large foreign-currency-denominated debt, however, exchange rate depreciation can quickly inflate the debt stock, creating a feedback loop that undermines sustainability (Calvo et al., 2003). This structural impediment to debt management in an economy such as Pakistan is the so-called original sin problem, the inability of many developing countries to borrow in their own currencies (Eichengreen et al., 2003). Fiscal multipliers also affect sustainability by influencing the denominator of the debt-to-GDP ratio. Blanchard and Leigh (2013) established that, in instances of fiscal tightening in the post-crisis euro zone, multipliers were much larger than previously thought; that is, austerity policies designed to reduce debt ratios actually increased them by lowering GDP more severely than projected. This observation has far reached consequences to the design of the fiscal adjustment programs in highly indebted developing countries, such as Pakistan, which has repeatedly undertaken IMF-supported programs of fiscal adjustment Haque and Montiel, (1991); Hussain, (2009).

Another body of parallel literature focuses on the institutional determinants of debt sustainability, and asserts that the quality of fiscal institutions, i.e., budget rules, independent fiscal councils, transparency mechanisms, and debt management offices is as important as the macroeconomic fundamentals. The early theoretical underpinnings were presented by Alesina and Tabellini (1990), who showed that political polarization could result in excessive debt accumulation, since rival governments would engage in strategic debt issuance to restrain future administrations. This model can be empirically relevant in politically divided societies in which short-term political gain is often traded off against fiscal responsibility. In a review conducted by the IMF (2013), countries with stronger fiscal structures - in terms of numerical fiscal rules, medium-term expenditure arrangements, and autonomous fiscal control - had a lower deficit bias and more sustainable debt trajectories. Schaechter et al. (2012) listed fiscal rules in 81 countries and found that fiscal rules on debt ceilings, balanced budget requirements, and expenditure limits were all associated with better fiscal discipline, but the quality of enforcement substantially moderated the effects. The design requirements for effective fiscal rules were previously stated by Kopits and Symansky (1998), who believed that such rules need to be simple, flexible, strongly enforced, and based on transparent accounting frameworks. Wolswijk and de Haan (2005) in Europe and Wheeler (2004) in general have discussed the role of debt management institutions. Both stress the significance of strategic management of debt- managing the composition, maturity structure and currency denomination of public debt in order to minimise rollover risks and cost of

debt. Rollover risk is an important weakness in countries with thin domestic capital markets that heavily depend on short-term debt financing their operations. Corruption and institutional weakness exacerbate the debt sustainability problem by inflating borrowing costs, reducing the quality of public expenditure, and eroding the revenue base through tax evasion and avoidance. Mauro (1995) recorded that corruption is linked to low levels of investment and low growth rates, whereas Gupta et al. (2001) documented that weak governance is related to high levels of military expenditure and low levels of social expenditure, further distorting the fiscal allocations towards growth-enhancing uses. These processes have a direct relationship with the sustainability challenge of debt in Pakistan Chaudhry et al., (2020); Javid et al., (2011).

The evidence on the sustainability of debt in emerging market economies (EMEs) points to structural characteristics that make EMEs and developing economies unique compared with advanced economies. The lack of domestic capital market depth, heavy reliance on commodity exports, unstable remittance flows, vulnerability to sudden stops in capital inflows, and weak institutional capacity are all contributors to the debt vulnerabilities Calvo et al., (2003); Reinhart et al., (2003). The joint DSF of LICs created by the World Bank and IMF in 2005, revised in 2012 and 2017, is the most systematic effort to operationalize these country-specific factors into sustainability measures. Cordella et al. (2010) criticized previous versions of the DSF, arguing that the mechanical implementation of uniform debt thresholds does not adequately account for country-specific institutional quality and export volatility. They interpreted the findings to mean that a country with a better policy environment can maintain higher debt levels without falling into distress. This result can be linked to the general argument by Acemoglu et al. (2003) that institutions are central to determining long-run economic outcomes, such as fiscal capacity. The experience of African debt has created abundant case study material of the dynamics of sustainability. Investigating the effect of the HIPC Initiative on debt relations in sub-Saharan Africa, Kasekende and Atingi-Ego (2010) found that debt relief, when coupled with improved fiscal management, created fiscal space for poverty-reducing expenditures. Nonetheless, Gueye and Sy (2015) cautioned that a rapid re-accumulation of debt following the completion of HIPCs risked reversing these gains, a trend observed in some post-HIPC countries, which in turn have accessed the Eurobond markets at commercial rates. The debt crisis in Latin America provides further comparative information. Calvo et al. (1993) reported the dynamics of capital account liberalisation and sudden stops that caused the debt crises in Argentina, Brazil, and Mexico, and

Edwards (2003) analysed under what conditions sovereign debt restructurings were successful or failed to restore sustainability. The general similarity between these experiences lies in the fact that it is not only the level of debt that determines sustainability, but rather the combination of debt structure, institutional quality, external financing terms and political will to bring about adjustment.

The issues of debt sustainability in Pakistan are rooted in structural fiscal imbalances that have persisted since independence. The country has been experiencing persistent fiscal deficits, funded by a combination of domestic bank and non-bank borrowing and external borrowing from bilateral creditors, multilateral institutions, and, more recently, commercial sources such as Eurobonds and Chinese loans under the CPEC. Arby and Hussain (2012) documented how the public debt of the government of Pakistan has changed over the years since the government began incurring debt in the 1990s, reaching more than 90% in the early 2000s. The tax-to-GDP ratio in Pakistan (one of the lowest in the world at around 10-11%), may be seen as perhaps the most fundamental structural constraint in fiscal sustainability Chaudhry et al., (2020); Kemal, (2001). By comparison, South Asia has an average tax ratio of around 13-15%, while most OECD countries collect 30-40% of GDP in taxes. This loss of revenue which is based on a small tax base, a high level of exemptions, low administrative capacity and political opposition to taxing agriculture and real estate, makes the government have to borrow money to cover even basic spending. Malik (2016) estimated the potential tax revenue of Pakistan, assuming that the collection is optimized, to be 16-20% of the GDP, which points to the fact that structural tax reform is a precondition to long-term debt sustainability.

External debt is a significant liability in Pakistan's debt profile. In their study, Hyder and Shah (2004) examined the pass-through of exchange rates in Pakistan and found that currency depreciation had a rapid and significant effect on the rupee price of dollar-denominated external liabilities. This effect of a currency mismatch was tremendously demonstrated during the sharp depreciation of the rupee in 2018-2019 and again in 2022-2023, when the exchange rate fell between approximately PKR 115/USD to PKR 285/USD, which caused the external debt stock measured in rupees to nearly double without any further borrowing. Husain (2009) examined the composition and sustainability of Pakistan's external debt, with a special focus on the increased share of bilateral debt from China, Saudi Arabia, and other creditors, as well as traditional multilateral obligations to the IMF, World Bank, and Asian Development Bank (ADB). Financing

relating to CPEC has received specific academic and policy interest. Hurley et al. (2018) evaluated CPEC debt risks in the broader context of China's BRI, finding that Pakistan was among the nations at high risk of debt distress from BRI-related borrowing, given the combination of project-related debt service obligations and existing macroeconomic imbalances. Nevertheless, Khalid et al. (2019) provided a more nuanced perspective, arguing that an increase in infrastructure investment under CPEC could boost growth and export capacity in the face of governance challenges, which were at the core of the dynamics of debt sustainability.

Pakistan has accessed IMF-supported programs on many occasions, being the world's most frequent user of IMF facilities, with 19 programs since 1958. All the programs have implied fiscal adjustment conditionality to lessen primary deficits, raise revenues and rationalise expenditure. In the 1980s, Haque and Montiel (1991) analysed the effectiveness of IMF programs in Pakistan during the 1980s, and found mixed evidence on the success of IMF programs in restoring fiscal sustainability in Pakistan or merely delaying underlying structural problems. More recent IMF Debt Sustainability Analyses of Pakistan (IMF, 2019, 2023) have used the MAC DSA framework, which exposes the economy's underlying projections to standard shock scenarios such as exchange rate, growth, and interest rate shocks. These ratings have consistently assessed Pakistan as being at high risk of debt distress when unfavourable conditions arise, although the base-case scenario under the program indicates gradual debt stabilisation. Chaudhry et al. (2020) critically reviewed these IMF assessments, stating that they are often overly optimistic about revenue mobilisation and growth prospects, thereby understating the actual sustainability risks. They also emphasised the pro-cyclical nature of fiscal adjustment under IMF conditionality, which can exacerbate recessions and jeopardise the revenue recovery on which the programs rely. To determine the degree to which the government of Pakistan has in the past responded to increasing debt levels by enhancing primary balances, Javid et al. (2011) adopted a fiscal reaction function approach that follows Bohn (1998) in measuring how Pakistan has in the past responded to increasing debt levels by improving primary balances. The results were sobering: Pakistan's fiscal response function showed weak or statistically insignificant adjustment, suggesting that the intertemporal solvency condition has not been systematically upheld. This fiscal insensitivity contrasts with the hypothetical nature of sustainable debt dynamic commitments and casts doubt on the credibility of the long-term commitment to fiscal consolidation.

The relationship linking the elevated debt levels in Pakistan and macroeconomic performance has been explored in several studies. Ahmed et al. (2012) examined the debt-growth relationship using time-series data for Pakistan and found evidence of a negative relationship, indicating that high levels of external debt have a negative impact on growth by crowding out private investment. A stark example of how debt accumulation can crowd out development-oriented spending on education, health, and infrastructure is Pakistan's debt service burden, which at one point or another has consumed 40-50% of fiscal revenues. Siddiqui and Malik (2001) analysed Pakistan's debt Laffer curve and tested the hypothesis that the country was in an area where a reduction in debt would trigger an increase in output to a level sufficient to sustain debt service capacity. Their analysis indicated that Pakistan had excessive external debt, above the optimal level, suggesting that debt relief or restructuring may be necessary to revive growth-enhancing investment levels. Abbas and Christensen (2010), in a broader cross-country study, found this conclusion echoed: at moderate levels of domestic debt, it may facilitate financial development and support growth, but at high levels of domestic debt, it crowds out private credit and dampens investment.

Domestic debt management in Pakistan has undergone a major transformation in recent decades. In their study, Khalid and Guan (1999) investigated the connection between monetary policy and domestic debt in Pakistan and found that monetary policy was dominated by the fiscal authority's financing needs. This trend, whereby the State Bank of Pakistan (SBP) monetises government debt to avoid crowding out the private sector, has inflationary implications and can undermine the real value of government debt obligations and macroeconomic stability. The development of Pakistan Investment Bonds (PIBs) and Treasury Bills (T-Bills) markets has been a significant aspect of debt management reform. Siddiqui (2008) reported an increase in the depth of the domestic debt market and an extension of maturities following the financial sector reforms in the 2000s, which decreased rollover risk by increasing the average maturity of domestic borrowing. Cheema and Malik (2020), however, observed that the management of domestic debt still faces problems due to underdeveloped secondary markets, a lack of institutional investor participation beyond the banking sector, and the high concentration of domestic debt in commercial banks, which creates systemic vulnerability.

The comparative analysis situates Pakistan's debt issues within a regional context. Comparing fiscal sustainability across South Asian economies: India, Pakistan, Bangladesh and

Sri Lanka, Abbas et al. (2011) find that despite India benefiting due to stronger growth to stabilise its debt, Sri Lanka and Pakistan face more acute sustainability pressures due to a combination of low revenue mobilisation, high cost in serving its debt and more volatile growth trajectories. The sovereign default of Sri Lanka in 2022 has heightened awareness of the prevalent weaknesses in South Asian economies, including the accumulation of external debt, the depletion of foreign exchange reserves, and delayed fiscal adjustment. Abbas et al. (2011) have created a historical database of government debt on a large sample of countries, which allows cross-country comparison of debt accumulation episodes. Their analysis has shown that a debt crisis is often preceded by a period of rapid accumulation of debt fueled by fiscal deficits, external shocks, and currency depreciation, a trend that is clearly visible in Pakistan's debt history. The global comparison also highlights that Pakistan's debt-to-GDP ratio is quite high by regional standards, but it is not unique in the world, which is why the ultimate test of sustainability is the country's ability and political will to make the necessary adjustments.

The literature review has presented a wide and heterogeneous range of scholars' opinions on the issue of debt sustainability, with special focus on the fiscal problems in Pakistan. Yet several important research gaps remain. While the macroeconomic determinants of debt sustainability have been extensively studied, the literature does not examine the major element of debt sustainability, such as the primary expenditure gap, the gap between the actual primary expenditure and the estimated amount of primary expenditure and its consistency with debt sustainability. This gap is especially noteworthy in Pakistan, where fiscal imbalances are primarily expenditure-related. The lack of information about the primary expenditure gap prevents policymakers from designing effective fiscal consolidation measures. Lack of knowledge about the sustainability of government expenditure can lead to policy responses that focus on short-term deficit reduction rather than long-term fiscal stability. Thus, the main goal of the paper is to assess Pakistan's debt sustainability using a fiscal response function that includes key macroeconomic and political variables, such as the primary balance, total public debt, the GDP gap, and a democracy dummy. More to the point, the research notes that primary expenditure is a gap in the current literature and needs to be addressed by integrating expenditure dynamics into the analysis of fiscal sustainability.

1. DEBT SUSTAINABILITY THEORETICAL FRAMEWORK

"The IMF and World Bank, broadly, define the "sustainability of debt" as a country's capacity to meet its debt service obligations in the present and the future without resorting to debt rescheduling or arrears or threatening to slow its economic growth" (IMF and World Bank 2013).

The baseline macroeconomic debt sustainability analysis (DSA) is about projection of different macroeconomic variables, yearly 4% medium-term real GDP growth, around 3% yearly CPI inflation over the long-term, to reduce current account deficit from 9.5% to 5% of GDP, by increasing exports and import substitution, an immediate increase in remittances, primary surplus by fiscal consolidation is required, external financing is assumed grants and concessional loans[1]

The quality of a DSA is critically dependent on the macroeconomic forecasts that form the basis of the projections of the future growth of the economy and how it reacts to external factors (in particular the outlook of financial markets) and to policy action. (2015). But from this perspective, the IMF has been found to be weak, particularly as a stabilizer in the case of economies in crisis. The problem seems twofold: in the evaluation of growth possibilities, and the analysis of policy alternatives (sometimes leading to advice or conditionality). In general the results suggest that institutional quality appears to be the most significant factor of fiscal performance across the selected countries, and that the fiscal adjustment response of the debt variable is not strong enough. The Hausman test results support the use of the random effects approach to analyze the behavior of fiscal response of the economies in the sample during the 2002-2024 period.

According to Uctum, M., & Wickens, M. (1997), Bohn, H. (1998), Arnone, M., & Presbitero, A. F. (2007), Contessi, S. (2012), Tielens, J., et al (2014), Mahmood, T., et al (2014), Adler, G., & Sosa, S. (2016), Kaur, B., et al (2018), and Chandia, K. E., et al (2019), a debt position is considered sustainable if the government satisfies the **IBC**, which stipulates that the current market value of public liabilities must be equal to the discounted sum of expected future primary surpluses. The intertemporal annual government budget constraints that is expenditures (i.e., government purchases, (G_t) plus interest on outstanding debt $(i_t D_{t-1})$ must be funded by three

taxes (T_t), new borrowing from the private sector ($\Delta D_t = D_t - D_{t-1}$), and changes in the stock of money ($\Delta M_t = M_t - M_{t-1}$).

$$G_t + i_t D_{t-1} = T_t + (D_t - D_{t-1}) + (M_t - M_{t-1}) \qquad Equ\ (1)$$

` If tax receipts (T_t) are more than current government expenditures, (G_t) then the government enjoys a primary surplus; conversely, if current government expenditures exceed tax receipts, then the government runs a primary deficit. The term "primary" refers to the current flow of taxes and government spending, which includes both long-term structural spending of government (such as development and non-development expenditures) and cyclical components (such as transfer payment, pension, etc).

The primary surplus (or deficit) less interest payments on outstanding debt is defined as an overall (including interest) fiscal surplus if positive and an overall fiscal deficit if negative. In more general terms we can define the primary balance as: $S_t = G_t - T_t$ if $S_t > 0$ it means that is a primary deficit. The government budget constraint can be rearranged to reveal that the difference between government debt at the start and end of the year is equal to three parts: interest paid on government debt, the primary deficit, and the change in money supply. Increased interest rates (how much the government has to pay on its debt), outlays, and decreased tax revenues will increase a country's debt. The intertemporal budget constraint (IBC) for the government is given as,

$$D_t = (1 + i_t)D_{t-1} + S_t - \Delta M_t \qquad equ\ (2)$$

$$D_t = D_{t-1} + (S_t + i_t D_{t-1}) - \Delta M_t \qquad equ\ (3)$$

Equation (3) explains that total debt D_t is equal to previous debt D_{t-1} plus overall fiscal deficit $s_t + i_t D_{t-1}$ minus the change in money / seigniorage. Equation (2) is divided by nominal GDP $P_t Y_t$ and simplified to get the debt to GDP ratio d_t equation (9) which concludes that the ratio depends on debt to primary balance s_t change in real money balance m_t ratio of previous debt d_{t-1} real GDP growth and real interest rate.

$$\frac{D_t}{P_t Y_t} = (1 + i_t)\frac{D_{t-1}}{P_t Y_t} + \frac{S_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t} \qquad equ\ (4)$$

$$\frac{D_t}{P_t Y_t} = (1 + i_t)\frac{P_{t-1}Y_{t-1}}{P_t Y_t}\frac{D_{t-1}}{P_{t-1}Y_{t-1}} + \frac{S_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t} \qquad equ\ (5)$$

$$d_t = (1 + i_t)\frac{P_{t-1}Y_{t-1}}{P_tY_t}d_{t-1} + s_t - \Delta m_t \qquad equ\ (6)$$

$$d_t = \frac{(1 + i_t)}{(1 + g_t)(1 + \pi_t)}d_{t-1} + s_t - \Delta m_t \qquad equ\ (7)$$

$$(1 + i_t) = (1 + r_t)(1 + \pi_t) \qquad equ\ (8)$$

$$d_t = \frac{(1 + r_t)}{(1 + g_t)}d_{t-1} + s_t - \Delta m_t \qquad equ\ (9)$$

$$\frac{(1 + r_t)}{(1 + g_t)} = \theta \ \ where \ \ \Delta m_t = 0$$

Here we assume that change in real money balance $\Delta m_t = 0$. Where most of the time $\theta$is called discount factor. Further, θ is substituted in equation (10) and recursively solved as follows. Where if $\frac{(1+r_t)}{(1+g_t)} = \theta$ is greater than 1 it means interest rate is more than GDP Growth, and vice versa; It concludes that debt to GDP ratio will persistently increase until the primary balance s_t counter balances the gap between interest rate and GDP growth.

$$d_1 = \theta_1 d_0 + s_1 \qquad equ\ (10)$$

$$d_2 = \theta_2 d_1 + s_2 \qquad equ\ (11)$$

$$= \theta_2[\theta_1 d_0 + s_1] + s_2$$

$$d_2 = \theta_2\theta_1 d_0 + \theta_2 s_1 + s_2 \qquad equ\ (12)$$

$$d_3 = \theta_3\theta_2\theta_1 d_0 + \theta_3\theta_2 s_1 + \theta_3 s_2 + s_3 \qquad equ\ (13)$$

$$d_4 = \cdots \ldots \ldots . so\ on$$

$$d_t = d_0 \prod_{i=1}^{t} \theta_i + \sum_{i=1}^{t} s_i \prod_{j=i+1}^{t} \theta_j \qquad equ\ (14)$$

$$d_t = d_0\theta^t + s\sum_{i=0}^{t1} \theta^t \qquad equ\ (15)$$

According to Bohn, H. (1998), Arnone, M., & Presbitero, A. F. (2007), Mahmood, T., et al (2014), Bouabdallah, O., et al (2017), the conventional debt accumulation approach, the debt-

to-GDP ratio changes are calculated from the interest rate-growth rate gap$(i_t - g_t)$, the primary balance (pb_t) and from one-off "deficit-debt adjustments" (dda_t), such as financial sector bail-outs or privatisation receipts Cuerpo, C et.al (2015). Future deterministic trajectories of these underlying factors are simulated and used to gain insights into the debt dynamics of these factors.

$$\Delta \mathrm{d}_t = \frac{i_t - g_t}{1 + g_t} d_{t-1} - pb_t + dda_t$$

While, Contessi, S. (2012), Mahmood, T., et al (2014), Malik, B. A. (2015), indicated necessary and sufficient conditions for debt sustainability.

1. The necessary condition for debt sustainability is $(\Delta \mathrm{d}_t = 0)$ and $\mathrm{r}_t < g_t$ real interest rate is less than GDP growth vice versa.
2. The sufficient condition for debt sustainability is $(\Delta \mathrm{d}_t = 0)$ and have on average positive primary surplus.

2. ECONOMETRIC METHDLOGY OF FISCAL RESPONSE FUNCTION, & A DEBT SUSTAINIBILITY ANALYSIS

This section of the study focuses to analyze the econometric methods which had been used to obtain long-run and short-run coefficients of fiscal response function and forecast of the debt sustainability. This paper employed ARDL econometric model to obtain long-run and short-run coefficients. Whereas Ejaz, M., & Hyder, K. (2019), estimated fiscal response function through OLS, 2SLS, GMM, and LIML but the direction of results is same as this study found. Actually this research employed ARDL because, it has superiority that variables are level and first difference, or a mix of the both. Like as in this case some variables are stationary at I (0), and others are at I (1). ARDL is not valid for second difference series and ARDL is valid for a small sample size. The Pesaran et al. (2001), initially proposed ARDL. The econometric regression for this paper is as follows,

$$\mathrm{PB} = \alpha_0 + \alpha_1 \mathrm{TPD} + \alpha_2 \mathrm{PE\_C} + \alpha_3 \mathrm{GDP\_C} + \alpha_4 \mathrm{DEM} + \mu_t \qquad (1)$$

In regression 1, PB represents primary balance, TPD, PE_C, and GDP_C, (total public debt, primary expenditure gap, and output gap, respectively) represents independent variables of fiscal response function. Control variables used in this regression is dummy of democracy (DEM) and μ_t is represent error term of FRF. The policymakers' targets a stable or mean-reverting debt path, the primary balance must modify positively to higher inherited debt Bohn, (1998). The

primary expenditure gap (PE_C), serves two theoretical roles. 1st, in models of fiscal rules and expenditure smoothing, deviations from a justified path (due to temporary windfalls or shocks) are corrected over time to preserve long-run solvency and intergenerational equity Debrun & Kumar, (2007); Barro, (1979). 2nd, the literature consolidation that the expenditure-based adjustments are often more durable and growth-friendly than revenue-based adjustments Alesina, et.al (2019); Alesina & Perotti, (1995). The GDP gap in the FRF helps separate cyclical from structural adjustments, aligning the estimated coefficient with long-run sustainability rather than cyclical co-movement Ilzetzki & Végh, (2008); Ilzetzki, et.al, (2013).

The democratic government make commitment, create accountability, and provide competitive environment through which democracy affects fiscal behavior Ali, S, et.al (2022). The democratic institutions enriches transparency, accountability, and checks & balances, which reduces rent-seekers and fiscal rule Ali, S., et.al (2022); Acemoglu, et.al (2019); Persson & Tabellini, (2003). Democracies are also more likely to adopt medium-term fiscal frameworks and independent fiscal councils, improving debt reactions Brender & Drazen, (2005); Rogoff, (1990).

3. ARDL ECONOMETRIC MODEL

The long-run and short-run coefficients of fiscal response function is obtained from following ARDL econometric model.

$$\Delta PB = \sigma_0 + \sum_{n=1}^{n} a_1 \Delta PB_{t-i} + \sum_{n=1}^{n} a_2 \Delta TPD_{t-i} + \sum_{n=1}^{n} a_3 \Delta PE\_C_{t-i} + \sum_{n=1}^{n} a_4 \Delta GDP\_C_{t-i} + \sum_{n=1}^{n} a_5 \Delta DEM_{t-i} + \beta_2 TPD_{t-i} + \beta_3 PE\_C_{t-i} + \beta_4 GDP\_C_{t-i} + \beta_5 DEM_{t-i} + \varepsilon_t \qquad Model\ 1$$

Where, Δ represents first difference operator, σ_0 represents intercept, ε_t represents white noise error term. Whereas α and β slope coefficients from the econometric model represent the short-run and the long-run dynamics respectively.

The other part of this papers methodology is to forecast either debt is sustainable or not. The uncertainty problem prevails whenever economic indicators are forecasted. In this paper total public debt percentage of GDP, domestic debt percentage of GDP, and external debt percentage

of GDP are forecasted unilaterally through ARMA econometric model. The ARMA econometric models are written as fallow for total public debt, domestic debt, and external debt.

$$TPD_t = \alpha_0 + \alpha_1 TPD_{t-1} + \cdots \ldots + \alpha_p TPD_{t-p} + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots \ldots + \theta_q \varepsilon_{t-q} \qquad Model\ 2$$

$$DD_t = \beta_0 + \beta_1 DD_{t-1} + \cdots \ldots + \beta_p DD_{t-p} + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots \ldots + \theta_q \varepsilon_{t-q} \qquad Model\ 3$$

$$ED_t = \gamma_0 + \gamma_1 ED_{t-1} + \cdots \ldots + \gamma_p ED_{t-p} + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots \ldots + \theta_q \varepsilon_{t-q} \qquad Model\ 4$$

The model two, three and four are showing that the total public debt, domestic debt, and external debt are depending on its several lags and error term of the model and error term lags.

Finally this paper used fan chart to visualize most likely, path of the point estimates with entire probability occurrence rang of the forecasted points. In the literature different economic variables are showed through fan chart. Ahmed, R., et.al (2021) forecasted house price sustainability through fan chart approach. Ejaz, M., & Hyder, K. (2019), Bouabdallah, O., et.al (2017), Cuerpo, C., et.al (2015) and other scholar had used fan chart approach for debt sustainability analysis. This study therefore used fan chart to visualize the point estimate with entire probability occurrence rang of forecasted points.

DATA SOURCES

Table 4.1: Data Sources

VARIABLE	ABR	SOURCES
Primary Balance	PB	Different Economic Surveys of Pakistan from 1983-84 to 2023-24
Total Public Debt	TPD	Different Economic Surveys of Pakistan from 2021-22 and 2023-24
Domestic Debt	DD	Different Economic Surveys of Pakistan from 1983-84 to 2023-24
External Debt	ED	Different Economic Surveys of Pakistan from 1983-84 to 2023-24
Gross Domestic Product	GDP	SBP
Primary Expenditure	PE	SBP
GDP Gap / Output Gap	GDP_C	SBP Hodrick-Prescott Filter
Primary Expenditure Gap	PE_C	SBP Hodrick-Prescott Filter
Democracy	DEM	Dummy

DEFINITION / DISCRIPTION OF VARIABLES

All researchers try to establish proper theoretical association of variables in empirical outcomes. However, for some reason, it is not always possible to use the required variables: There

may not be enough time, funding, or data series may be unavailable. So, for that, researchers always used alternative variable or a proxy of unavailable series. The data for the democracy, the GDP gap and the Primary expenditure gap are not available across time in this study. To address this issue, study employed a Dummy variable for democracy (1 for democracy and 0 for else), and use Hodrick-Prescott Filter for GDP gap and Primary expenditure gap. Primary expenditure, total public debt, external debt, domestic debt were all expressed as percentages of GDP; while primary balance was measured as a percentage of total receipts. Finally the sample size of data is from 1976 to 2024.

THE ARDL RESULT OF FISCAL RESPONSE FUNCTION

This section of the article covers estimation results of fiscal response function through ARDL econometric model. Initially, it confirms the association of fiscal response function variables. Ejaz, M., & Hyder, K. (2019), estimated fiscal response function through OLS, 2SLS, GMM, and LIML but when data test for stationarity, it is found that some variables are stationary at I (0), and others are at I (1). Therefore this paper employed ARDL econometric model to obtain long-run and short-run coefficient. The table 5.1 shows the results of unit root through the ADF and PP test. There it could be seen that three variables PB, TPD, and GDP_C (Primary balance, Total public debt and GDP gap) are stationary at I (0) level, and two variables PE_C and DEM (Primary expenditure gap and Dummy for democracy) are stationarity at I (1) 1st difference.

When variables have mixed order for stationarity then ARDL model is an appropriate for estimation of short-run and long-run coefficient. Bound test is employed to identify the long-run association between fiscal response function variables. Table 5.2 shows the bound test results; it indicates the calculated F-statistic of 7.64 is greater than all the critical F-statistics levels. Therefore, it is established that there is a long-run association between fiscal response variables PB, TPD, PE_C, GDP_ and DEM (Primary balance, Total public debt, Primary expenditure gap, GDP gap, and Dummy for democracy).

Table No.5.1: The Unit Root Tests Outcomes through h Phillips & Perron and Augmented Dickey-Fuller Test of Variables Used for Fiscal Response Function

The Variable	Augmented Dickey Fuller with intercept				PP with intercept			
	Level		1st Difference		Level		1st Difference	
	T-Statistics	P-Statistics	T-Statistics	P-Statistics	T-Statistics	P-Statistics	T-Statistics	P-Statistics

PB	--2.78	0.06***	-9.02	0.00*	-2.78	0.06***	-9.06	0.00*
TPD	-3.00	0.04**	-10.68	0.00*	-2.98	0.04**	-10.68	0.00*
PE_C	-0.86	0.78	-8.55	0.00*	-4.07	0.00*	-10.50	0.00*
GDP_C	-6.72	0.00*	-3.48	0.01**	-2.59	0.10	-3.03	0.04**
DEM	-2.09	0.45	-6.70	0.00*	-2.23	0.19	-6.70	0.00*

Source: The author's computations. The significance indicated as 10% ***, 1%*, and 5%**

Table No.5.2: ARDL Bound Test Results of Fiscal Response Function

	Values	K
F-Value	7.64	4
Critical Bounds		
Significance Levels	l 0 Bound	L 1 Bound
10%	2.2	3.09
5%	2.56	3.49
2.5%	2.88	3.87
1%	3.29	4.37

Source: the author's computation

Furthermore, the long-run coefficients for the fiscal response function are calculated, and each variable's significance level and direction are given in Table 5.3. The TPD slope of 0.166 with P-statistic 0.005 indicates that TPD is positively and significantly associated with PB. Whereas, Ejaz, M., & Hyder, K. (2019), showed positive and significant slopes through OLS, 2SLS, GMM, and LIML. Therefore, it is concluded that an increase in debt leads to an increase in primary balance/ primary deficit. The PE_C long-run slope coefficient -0.009 with P-statistic 0.012 indicates that PE_C is negative and significantly associated with PB. The GDP_C long-run slope coefficient 0.0014 with P-statistics 0.006 indicates that the GDP_C is positive and significantly associated with PB. . The DEM long-run slope coefficient 3.143 with P-statistics 0.004 indicates that the DEM is positive and significantly associated with PB. The constant intercept C is -13.195 with P-statistics 0.003 negative and significantly associated with PB. The results are in-line with theory and literature like as Ejaz, M., & Hyder, K. (2019), Kaur, B, et al (2018), Bouabdallah, O, et al (2017), Cuerpo, C., & Ramos, J. M. (2015), Contessi, S. (2012), Mackiewicz, M. (2008), and Roubini, N. (2001).

The other task is to investigate the short-run association between variables, and it is done by ECM. The short-run behaviour of variables is displayed in Table 5.4. Lag of primary balance D (PB (-1)) and lag of total public debt D (TPD (-1)) have a negative and significant association with PB in the short-run. Whereas, in the short-run democracy D(DEM) is negatively and significant impact on PB. Finally the most important is the ECT coefficient is highly significant at 1% level and determine that the high speed of convergence to the long-run equilibrium. The ECT coefficient is -0.265, which is very significant. It calls for a high rate of short-run divergence from the long-run equilibrium in a year, corrected by 26%. The model lag length is determined according to the AIC and the lag length of models is shown on Figure 5.1. The selected model ARDL is (2, 2, 0, 0, 2).

Table No.5.3: The ARDL Long-run Slope Coefficient Results of Fiscal Response Function

	TPD	PE_C	GDP_C	DEM	C
Coefficients	0.166*	-0.009***	0.0014**	3.143*	-13.195*
Standard Errors	0.055	0.003	0.0004	1.029	4.167
T-Statistics	2.986	-2.643	2.898	3.052	-3.166
P-Statistics	0.005	0.012	0.006	0.004	0.003

Source: The author's computation. The significance indicated as 10% ***, 1%*, and 5% **

Table No.5.4: The Short-run Error Correcting Dynamics for Fiscal Response Function

	D(PB(-1))	D(TPD)	D(TPD (-1))	D(DB(-2))	D(DEM)	Coin Eq (-1)
Coefficients	-0.289	0.01	-0.038	0.402	-1.373	-0.265
Standard Error	0.100	0.011	0.011	0.445	0.449	0.036
T-statistics	-2.874	0.906	-3.256	0.901	-3.056	-7.227
P-statistics	0.006*	0.370	0.002*	0.373	0.004*	0.000*
R-Square	0.626			Log Likelihood	-56.486	
Adjusted Square	0.580			AIC	2.658	

S.E of Regression	0.861			SC	2.895	
Sum Squared Resid	30.445			HQC	2.747	

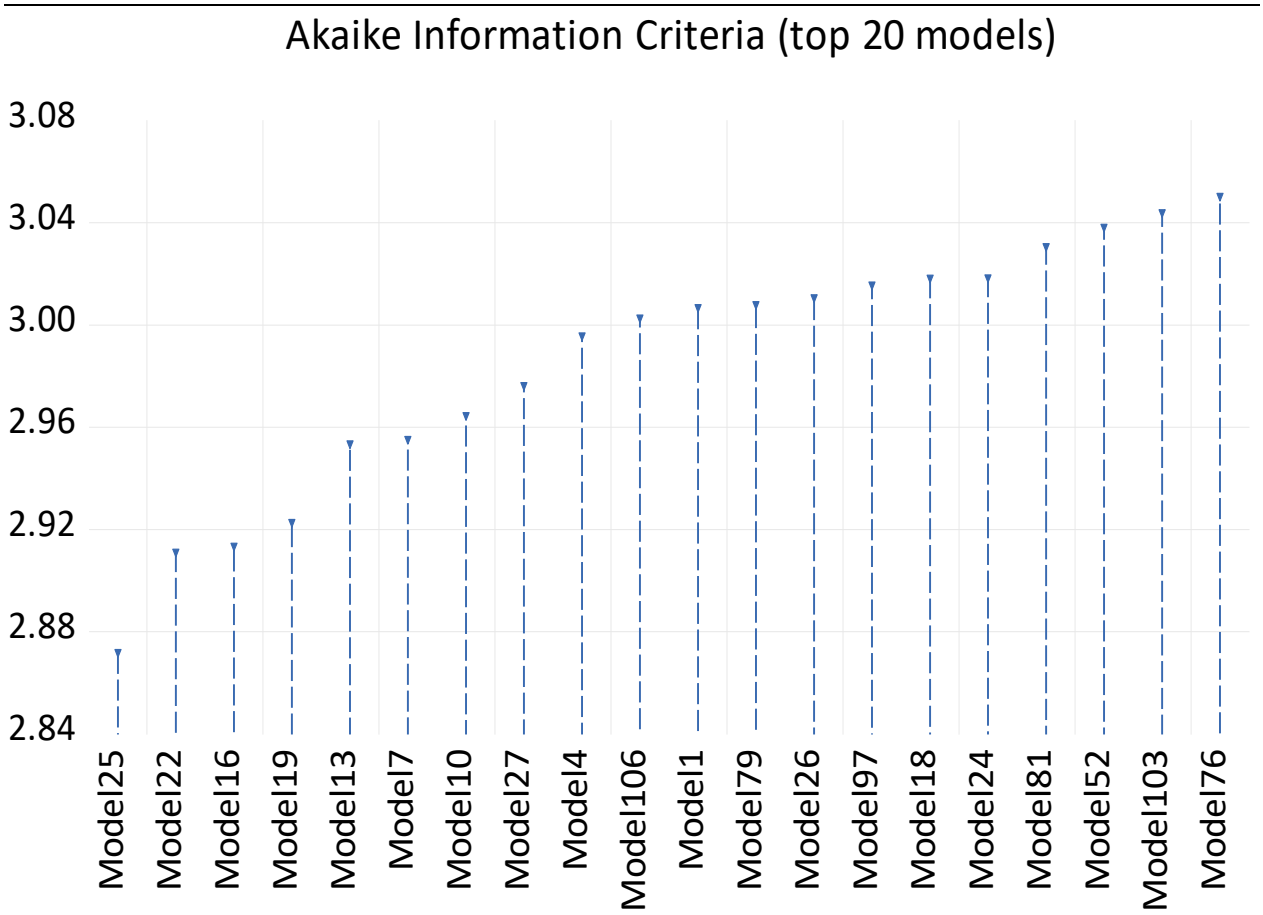
Source: The author's computation. The significance indicated as 10% ***,1%* and 5% **

Table No.5.5: Econometric Problem Deduction methods

Statistics	Heteroskedasticity Test: Breusch-Pagan-Godfrey	Breusch-Godfrey Serial Correlation LM Test:	Ramsey RESET Test
F	0.301	0.069	0.435
P	0.976	0.933	0.513

Source: The author's computation. The significance indicated as 10% ***,1%*, and 5% **

The results of Breusch-Pagan-Godfrey for heteroskedasticity, the serial correlation LM test conducted for autocorrelation and Ramsey RESET test conducted for model specification were indicated in Table 5.5. The P and F statistics of all of the tests are insignificant, which shows that the model does not have any heteroskedasticity, autocorrelation or model specification bias. The CUSUM and CUSUMSQ tests were used to test the stability of parameters (Pesaran & Pesaran, 1997). CUSUM tests were used to test regressors' and intercept structural break mean and CUSUMSQ tests were used to test the slope coefficient or variance of error term breaks (Turner, 2010). The parameters are stable as can be seen from the CUSUMQ and CUSUMS graphs, which are shown in Figure 5.3 and 5.2 correspondingly, and both of which lie within the 5% confidence interval region.



Model25: ARDL(2, 2, 0, 0, 2)
Model22: ARDL(2, 2, 0, 1, 2)
Model16: ARDL(2, 2, 1, 0, 2)
Model19: ARDL(2, 2, 0, 2, 2)
Model13: ARDL(2, 2, 1, 1, 2)
Model7: ARDL(2, 2, 2, 0, 2)
Model10: ARDL(2, 2, 1, 2, 2)
Model27: ARDL(2, 2, 0, 0, 0)
Model4: ARDL(2, 2, 2, 1, 2)
Model106: ARDL(1, 2, 0, 0, 2)
Model1: ARDL(2, 2, 2, 2, 2)
Model79: ARDL(2, 0, 0, 0, 2)
Model26: ARDL(2, 2, 0, 0, 1)
Model97: ARDL(1, 2, 1, 0, 2)
Model18: ARDL(2, 2, 1, 0, 0)
Model24: ARDL(2, 2, 0, 1, 0)
Model81: ARDL(2, 0, 0, 0, 0)
Model52: ARDL(2, 1, 0, 0, 2)
Model103: ARDL(1, 2, 0, 1, 2)
Model76: ARDL(2, 0, 0, 1, 2)

Figure: 5.1: Akaike Information Criteria

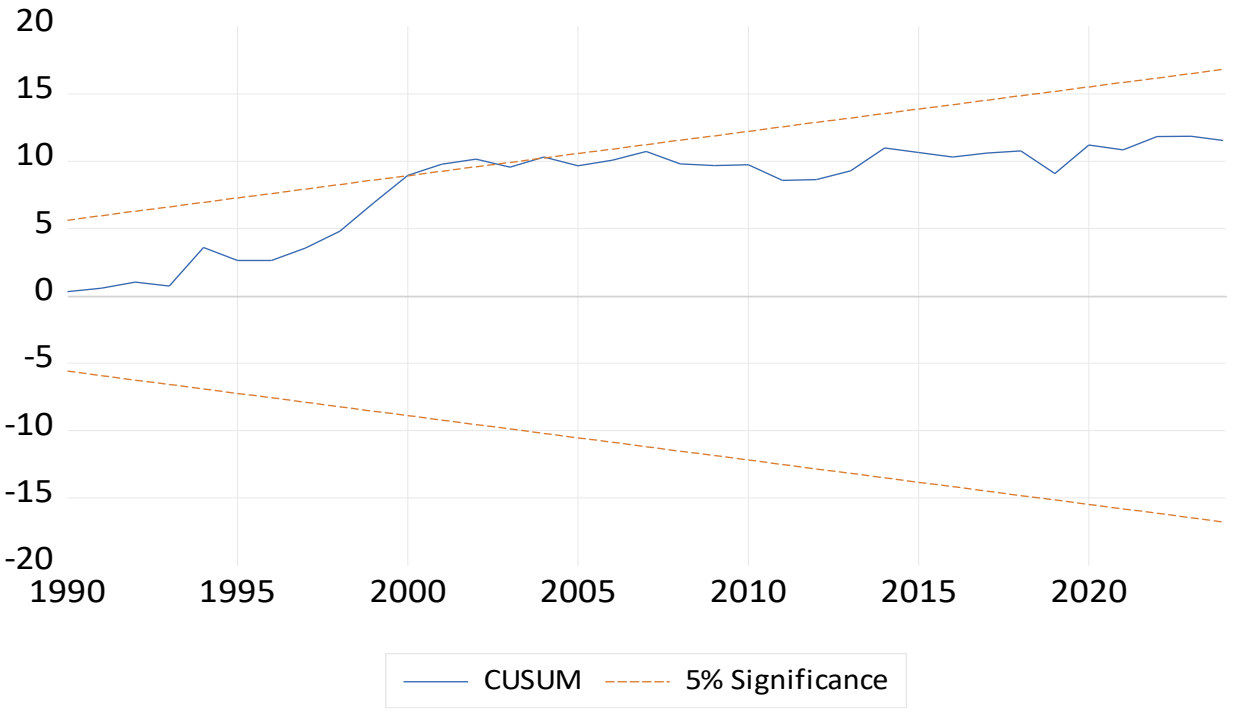


Figure 5.2: CUSUMS graphs

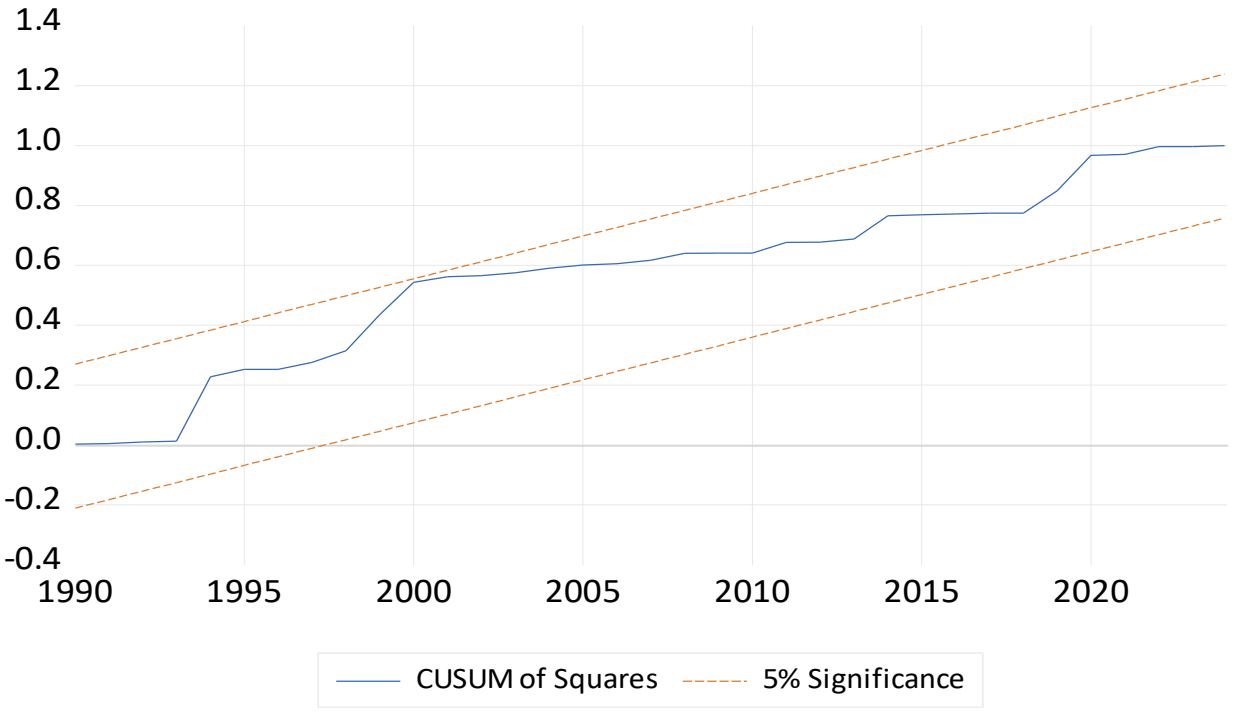


Figure 5.3: CUSUM of Square Graph

This section covers the results and its discussion of forecasting debt sustainability of three variables total public debt percentage of GDP (TPD), Domestic debt percentage of GDP (DD), and External Debt percentage of GDP (ED). The fan chart is valuable in the context of debt sustainability analysis as it provides an indication of uncertainty and risk in the debt trajectory, rather than a single trajectory. In fact, the state of the economy i-e GDP growth, interest rates, inflation, exchange rates, fiscal balances, and borrowing costs is unpredictable and can vary from one period to the next. A simple point forecast would be a forecast that assumes that the future will unfold in exactly one path—usually an unlikely path. The "fan chart" overcomes this weakness by showing a range of potential outcomes around the baseline debt forecast. Governments and policy makers are not only interested in the amount of public debt anticipated, but also in the risk that public debt may become unsustainable in bad economic scenarios in a debt sustainability study. The fan chart is a graphical representation of possible debt dynamics under varying degrees of uncertainty. The wider bands on the chart represent greater uncertainty of the forecast period. The wide bands indicate high fiscal sensitivity and macroeconomic volatility, while narrow bands indicate relatively stable and predictable debt dynamics.

The figure 5.4 and table 5.6 is showing the five years forecasted values total public debt as percentage of GDP. It is very interestingly all the five years forecasted value are lying in 30% confidence interval and trend of forecasted values are showing that the total public debt percentage of GDP is decreasing. Therefore it is concluded that total public debt is sustainable.

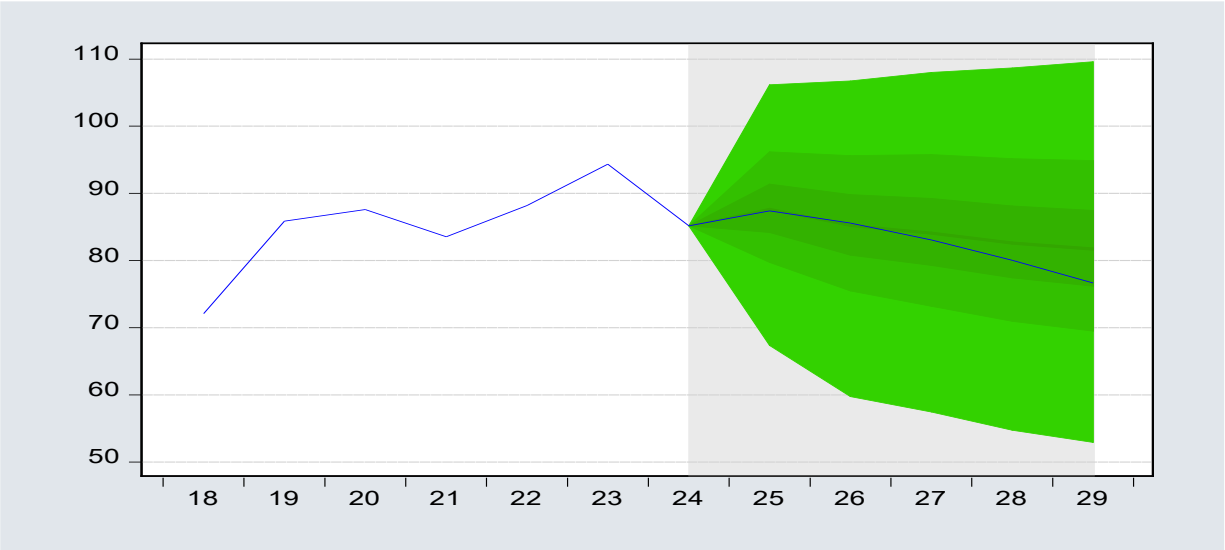


Figure: 5.4: Fan Chart of Total Public Debt Percentage of GDP. (Source the author computation)

Table No.5.6: Fan Chart Results of Total Public Debt Percentage of GDP

Year	TPD % GDP Forecasted	Confidence Intervals					
		Lower 30%	Upper 30%	Lower 60%	Upper 60%	Lower 90%	Upper 90%
2024	85.17	85.17	85.17	85.17	85.17	85.17	85.17
2025	87.40	84.16	91.37	79.71	96.19	67.36	106.15
2026	85.56	80.78	89.82	75.46	95.61	59.74	106.73
2027	83.07	79.26	89.23	73.17	95.76	57.43	107.99
2028	80.04	77.40	88.11	70.95	95.16	54.73	108.68
2029	76.63	76.18	87.44	69.48	94.88	52.91	109.58
Mean	71.89	STD. Dev	14.90	Skewness	-1.39	Kurtosis	7.63

Source the author computation.

The figure 5.5 and table 5.7 is showing the five years forecasted values domestic debt as percentage of GDP. Its results are totally different from total public debt that all the five years forecasted value are lying near to lower bound of 90% confidence interval and trend of forecasted values are showing that the domestic debt percentage of GDP is decreasing. Therefore it is concluded that the domestic debt is sustainable too.

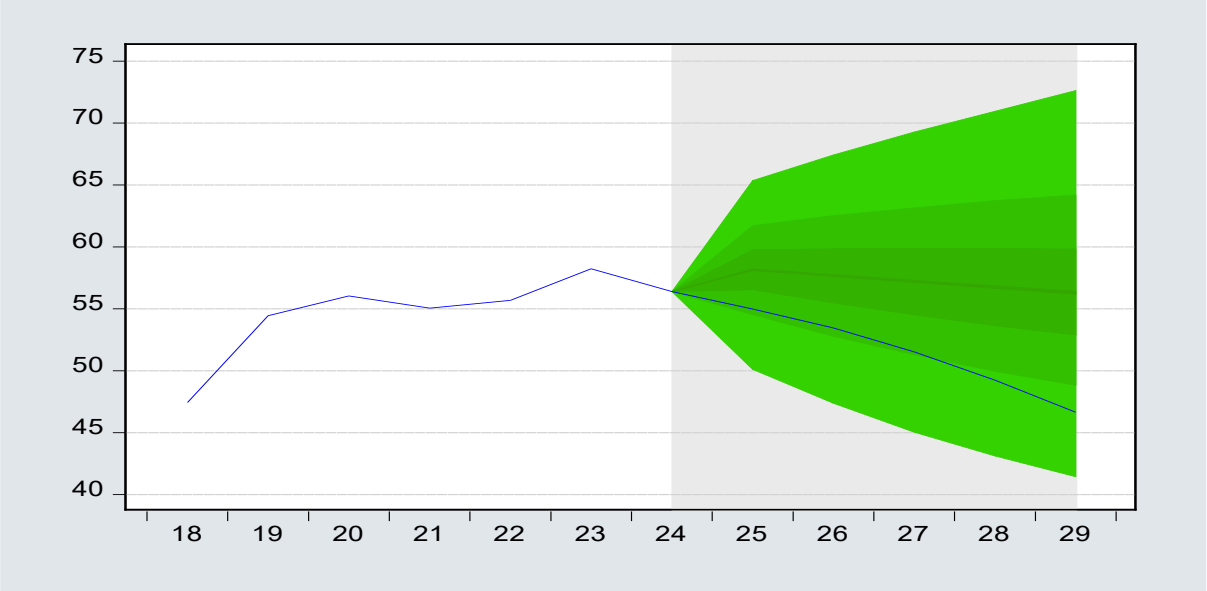


Figure: 5.5: Fan Chart of Domestic Debt Percentage of GDP (Source the author computation)

Table No.5.7: Fan Chart Results of Domestic Debt Percentage of GDP

Year	TPD % GDP Forecasted	Confidence Intervals					
		Lower 30%	Upper 30%	Lower 60%	Upper 60%	Lower 90%	Upper 90%
2024	56.37	56.37	56.37	56.37	56.37	56.37	56.37
2025	54.96	56.52	59.74	54.52	61.70	50.10	65.34
2026	53.44	55.47	59.84	52.78	62.51	47.37	67.39
2027	51.52	54.49	59.88	51.28	63.13	45.02	69.27
2028	49.23	53.63	59.85	49.95	63.73	43.11	70.93
2029	46.64	52.88	59.82	48.81	64.17	41.43	72.60
Mean	40.09	STD. Dev	10.30	Skewness	-0.15	Kurtosis	2.35

Source the author computation.

The figure 5.6 and table 5.8 is showing the five years forecasted values External debt as percentage of GDP. Its results are entirely different than above two variables total public debt and domestic debt but the five years forecasted value have too smaller differences and all values are lying near to lower bound of 90% confidence interval and trend of forecasted values are showing that the external debt percentage of GDP is decreasing. Therefore it is concluded that the domestic debt is sustainable too.

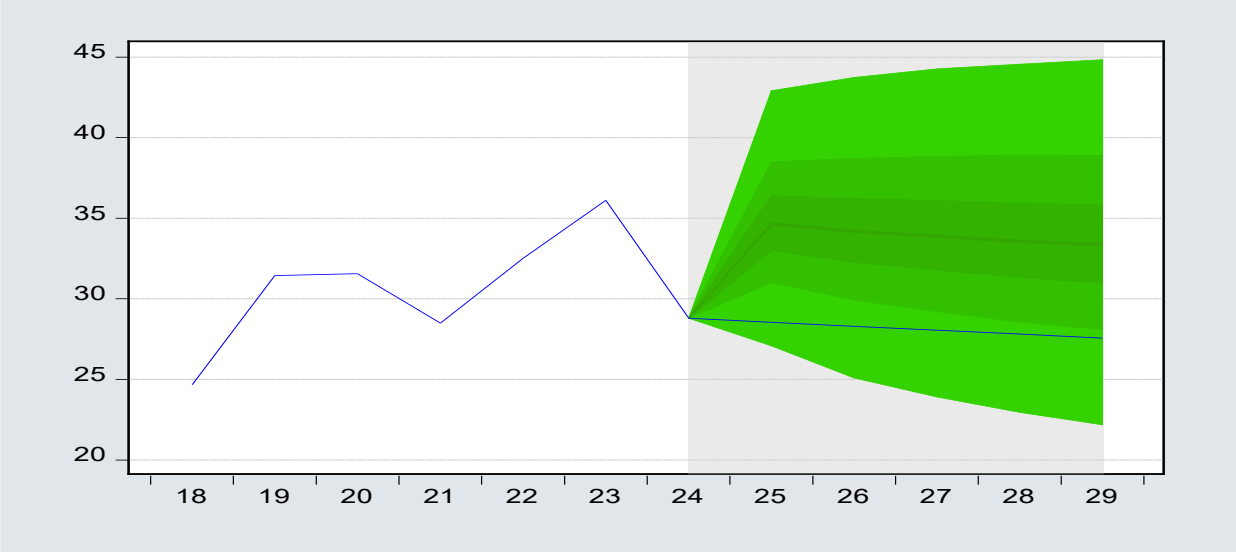


Figure: 5.6: Fan Chart of External Debt Percentage of GDP. (Source the author computation)

Table No.5.8: Fan Chart Results of External Debt Percentage of GDP

Year	TPD % GDP Forecasted	Confidence Intervals					
		Lower 30%	Upper 30%	Lower 60%	Upper 60%	Lower 90%	Upper 90%
2024	28.79	28.79	28.79	28.79	28.79	28.79	28.79
2025	28.54	33.00	36.39	31.01	38.50	27.08	42.90
2026	28.29	32.26	36.22	29.91	38.69	25.09	43.73
2027	28.05	31.78	36.10	29.02	38.82	23.90	44.25
2028	27.80	31.31	35.95	28.55	38.90	22.95	44.55
2029	27.56	30.97	35.81	28.08	38.89	22.18	44.83
Mean	32.35	STD. Dev	6.88	Skewness	0.21	Kurtosis	3.17

Source the author computation.

CONCLUSION

This study seeks to identify the debt sustainability dynamics of Pakistan through fiscal response function. Where the primary balance was modelled as function of total public debt, GDP gap, and democracy dummy, while focusing on the primary expenditure gap as a missing linkage in the existing literature. The empirical inspiration for the FRF approach lies in testing whether fiscal authorities respond to rising debt by improving the primary balance, and which is a necessary condition for long-term debt sustainability. Further this paper used modern fan chart to visualize most likely path of the point estimates with entire probability occurrence rang of the forecasted points.

The findings are confirmed by the literature and analytical framework that suggest Pakistan shows, at best**,** weak fiscal sustainability. While there is some proof that the primary balance reacts positively to rises in the public debt, the magnitude of this response remains insufficient to stabilize debt in the long run. Consistent primary deficits, together with increasing debt servicing obligations, continue to place a burden on the fiscal position. The inclusion of the output gap (GDP gap) highlights the cyclical nature of fiscal policy in Pakistan, where fiscal adjustments tend to be pro-cyclical rather than counter-cyclical, reflecting restricted fiscal space and structural inflexibilities.

The democracy dummy emphasizes the importance of political economy issues in shaping fiscal policy results. Democratic periods are often linked with high government expenditure because of electoral pressures, which may weaken fiscal policy discipline, whereas non-democratic regimes may demonstrate relatively strict fiscal control but absence of transparency and institutional accountability. These dynamics indicate that debt sustainability in Pakistan is not purely an economic issue but is also deeply influenced by institutional and political factors.

A key contribution of this study is the identification of a significant research gap in the existing literature: the absence of explicit consideration of the primary expenditure gap within the fiscal response function and the calculation of coefficient of it. While most studies focus on the primary balance as an aggregate indicator, they fail to distinguish between revenue and expenditure adjustments. In the context of Pakistan, this omission is particularly important because fiscal imbalances are largely expenditure-driven**.** The lack of attention to primary expenditure masks the underlying fiscal mechanisms and limits the effectiveness of policy recommendations. In the future, research should focus on how institution play its role in decreasing debt and improving economic growth and inflation in Pakistan. Therefore, a comprehensive understanding of debt sustainability requires moving beyond aggregate fiscal indicators and incorporating the behavior and composition of government spending and institutions.

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